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# QED Effective Action at Finite Temperature and Density

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## Abstract

The QED effective action at finite temperature and density is calculated to all orders in an external homogeneous and time-independent magnetic field in the weak coupling limit. The free energy, obtained explicitly, exhibit the expected de Haas – van Alphen oscillations. An effective coupling at finite temperature and density is derived in a closed form and is compared with renormalization group results.

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# 1 Introduction

Large magnetic fields are relevant in a number of physical systems like supernovas[1], where  $B = \mathcal{O}(10^{10})\text{T}$ , neutron stars [2], where  $B = \mathcal{O}(10^8)\text{T}$ , or white magnetic dwarfs [3] in which case  $B = \mathcal{O}(10^4)\text{T}$ . (As a reference the electron mass in units of tesla is  $m^2 = \mathcal{O}(10^9)\text{T}$ .) The radiative corrections to the magnetic moment of a Dirac fermion has been estimated in the presence of such large magnetic fields and it was argued that they are extremely small [4, 5]. It has recently been shown that a plasma at thermal equilibrium can sustain large fluctuations of the electromagnetic fields. For instance, in the primordial Big-Bang plasma, the amplitude of magnetic field (zero frequency) fluctuations at the time of the primordial nucleosynthesis can be as large as  $B = \mathcal{O}(10^{10})\text{T}$  [6]. Other systems with large magnetic fields present are mergers of massive black holes [7], where  $B = \mathcal{O}(10^{13})\text{T}$  or superconducting strings [8], where  $B = \mathcal{O}(10^{14})\text{T}$  or even larger. At the electroweak phase transition in the very early universe it has, furthermore, been argued that very large magnetic fields,  $B = \mathcal{O}(10^{19})\text{T}$ , can be generated due to gradients in the Higgs field [9].

In many of these systems one has to consider the effects of thermal environments. Calculation of the QED effective potential, i.e. the free energy, has been attempted before either at finite temperature [10, 11] or at finite chemical potential [12]. In the latter case the effective action is unfortunately not complete but the correct form is presented here. At finite chemical potential and for sufficiently small temperatures, the QED effective action should exhibit a certain periodic dependence of the the external field, i.e. the well-known de Haas – van Alphen oscillations in condensed matter physics. This was not obtained in Ref.[12].

Let  $\mathcal{L}_{eff}$  denote the QED effective action for a constant B-field at finite temperature  $T = 1/\beta$  and chemical potential  $\mu$ . We calculate this effective action to all orders in  $eB$  but with no virtual photons present, i.e. we consider the weak coupling limit. A more detailed analysis will be presented elsewhere [13].

## 2 Derivation of the Effective Action $\mathcal{L}_{eff}$

The basic relation we need in order to derive the one-loop correction to the effective Lagrangian is the identity

$$\frac{\partial \mathcal{L}_{eff}}{\partial m} = i \text{Tr } S_F(x; x) \quad , \quad (2.1)$$

where  $S_F(x; x')$  is the fermion propagator in the external magnetic field and the trace is over spinor indices. It can be constructed from the solutions of the Dirac equation  $(i\cancel{\partial} - e\cancel{A} - m)\psi(x) = 0$  in such a way that

$$S_F(x; x') = \langle 0 | \mathbf{T}[\psi(x) \bar{\psi}(x')] | 0 \rangle . \quad (2.2)$$

Equation (2.1) determines the vacuum part,  $\mathcal{L}_1 = \mathcal{L}_{eff}(T = \mu = 0, B)$ , of the effective action which should be added to the tree-level,  $\mathcal{L}_0 = -B^2/2$ . At finite temperature and density we simply replace the time-ordered vacuum expectation values in Eq.(2.2) by a thermal average. It can be shown that this replacement corresponds to the conventional calculational rules of thermo field dynamics. The solutions of the Dirac equation with an external constant magnetic field parallel to the  $z$ -axis are the standard relativistic Landau levels with energy spectrum given by

$$E_n(k_z) = \sqrt{m^2 + k_z^2 + 2eBn} , \quad (2.3)$$

where  $n = 0, 1, 2, \dots$  and  $k_z$  is the momentum parallel to the magnetic field. The construction of the propagator is similar to the zero temperature case [14] except that the propagating particles can now be exchanged with the heatbath. We find

$$\text{Tr } S_F(x; x) = \sum_{n=0}^{\infty} \int \frac{d\omega dk_y dk_z}{(2\pi)^3} [\Delta + (\Delta^* - \Delta) f_F(\omega)] 2m(I_n + I_{n-1}) , \quad (2.4)$$

where we have introduced the scalar propagator

$$\Delta = \frac{1}{\omega^2 - k_z^2 - m^2 - 2eBn + i\epsilon} . \quad (2.5)$$

The thermal distribution  $f_F(\omega)$  is given by

$$f_F(\omega) = \frac{\theta(\omega)}{e^{\beta(\omega-\mu)} + 1} + \frac{\theta(-\omega)}{e^{\beta(\mu-\omega)} + 1} , \quad (2.6)$$

and we use the notation

$$I_n = \left( \frac{eB}{\pi} \right)^{1/2} \exp \left[ -eB \left( x - \frac{k_y}{eB} \right)^2 \right] \frac{1}{n!} H_n^2 \left[ \sqrt{2eB} \left( x - \frac{k_y}{eB} \right) \right] , \quad (2.7)$$

where the functions  $H_n$  are Hermite polynomials, and we define  $I_{-1} = 0$ . From the propagator in Eq.(2.4) we get both the vacuum correction  $\mathcal{L}_1$  and a thermal correction  $\mathcal{L}_{eff}^{\beta, \mu}$ . It is well-known that a real-time formalism at finite temperature requires a doubling of the degrees of freedom and it can be shown that Eq.(2.4) is the 11-component of the

matrix propagator in thermo field dynamics [15]. Here we only need the 11-component for the one-loop calculation. The vacuum part of Eq.(2.4) that survives when  $f_F(\omega) \rightarrow 0$  reproduces the old result by Schwinger [16]

$$\mathcal{L}_1 = -\frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s^3} \exp(-m^2 s) \left( esB \coth(esB) - 1 - \frac{1}{3}(esB)^2 \right) . \quad (2.8)$$

Here  $\mathcal{L}_1$  has been renormalized by adding a second order polynomial in  $eB$ . We stress that the physics behind this renormalization is related to the fact that the coefficient in front of the quadratic term is proportional to the square of inverse (bare) coupling. This renormalization corresponds to a charge renormalization as well as a wave function renormalization in such a way that  $eB$  is invariant. This charge renormalization also leads to the weak coupling expansion of the QED  $\beta$ -function, i.e.

$$\lambda \frac{d}{d\lambda} \alpha(\lambda) = \beta(\alpha(\lambda)) = \frac{2}{3\pi} \alpha^2(\lambda) + \mathcal{O}(\alpha^3(\lambda)) , \quad (2.9)$$

where  $\lambda$  is a momentum scale factor. In order to calculate the thermal part  $\mathcal{L}_{eff}^{\beta,\mu}$  of the effective action, we have to be careful with the convergence and the analytical structure. We therefore let the sum over the quantum number  $n$  only go to a finite  $N$  and take the limit  $N \rightarrow \infty$  at the end. This gives

$$\text{Tr } S_F^{\beta,\mu} = \lim_{N \rightarrow \infty} i \frac{mB}{\pi^{3/2}} \text{Im} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_0^\infty \frac{ds}{s^{1/2}} e^{i\frac{3\pi}{4}} e^{-is(\omega^2 - m^2 - i\epsilon)} \left[ \frac{1 + e^{i2sB}}{1 - e^{i2sB}} - \frac{2e^{i2NsB}}{1 - e^{i2sB}} \right] . \quad (2.10)$$

The poles in the last factor cancel for finite  $N$ , and we cannot let  $N \rightarrow \infty$  in a naive way before deforming the  $s$  integration contour to the imaginary axis. After integrating Eq.(2.10) with respect to  $m$ , to get  $\mathcal{L}_{eff}^{\beta,\mu}$ , and being careful with the convergence when deforming the contours of integration we arrive at  $\mathcal{L}_{eff}^{\beta,\mu} = \mathcal{L}_0^{\beta,\mu} + \mathcal{L}_1^{\beta,\mu}$ , where

$$\mathcal{L}_0^{\beta,\mu} = \frac{1}{3\pi^2} \int_{-\infty}^{\infty} d\omega \theta(\omega^2 - m^2) f_F(\omega) (\omega^2 - m^2)^{3/2} , \quad (2.11)$$

is the ideal gas contribution in absence of the external field  $B$ , and

$$\begin{aligned} \mathcal{L}_1^{\beta,\mu} = & \int_{-\infty}^{\infty} d\omega \theta(\omega^2 - m^2) f_F(\omega) \left[ \frac{1}{4\pi^{5/2}} \int_0^\infty \frac{ds}{s^{5/2}} e^{-s(\omega^2 - m^2)} (seB \coth(seB) - 1) \right. \\ & \left. - \frac{1}{2\pi^3} \sum_{n=1}^{\infty} \left( \frac{eB}{n} \right)^{3/2} \sin \left( \frac{\pi}{4} - \frac{\pi n}{eB} (\omega^2 - m^2) \right) \right] . \end{aligned} \quad (2.12)$$

This is the main result of our paper. The term with the sum over  $n$  was neglected in Ref.[12] and we show in Section 3 that it is essential to keep this term in order to get the correct physical result.

The finite temperature part of the effective action is directly related to the free energy of a gas of relativistic fermions in a constant  $B$ -field. If  $Z(B, T, \mu)$  is the corresponding partition function, without the contribution from the thermal photon gas, we can also write

$$\begin{aligned} \mathcal{L}_{eff}^{\beta, \mu} = \frac{\log Z(B, T, \mu)}{\beta V} &= \frac{eB}{\beta(2\pi)^2} \sum_{\lambda=1}^2 \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dk \left\{ \log(1 + e^{-\beta(E_{\lambda, n} - \mu)}) \right. \\ &\quad \left. + \log(1 + e^{-\beta(E_{\lambda, n} + \mu)}) \right\}, \end{aligned} \quad (2.13)$$

where  $E_{\lambda, n} = \sqrt{m^2 + k^2 + 2eB(n + \lambda - 1)}$ , and  $\lambda$  labels the spin of the fermions. For  $|\mu| \leq m$ , Eq.(2.13) can be rewritten in the physically less transparent way

$$\mathcal{L}_{eff}^{\beta, \mu} = \frac{1}{(2\pi)^2} \sum_{l=1}^{\infty} (-1)^l \int_0^{\infty} \frac{ds}{s^3} \exp\left(-\frac{\beta^2 l^2}{4s} - m^2 s\right) eBs \coth(seB) \frac{\cosh(\beta l \mu)}{2}, \quad (2.14)$$

which for  $\mu = 0$  also is an equation given in [10]. However, it is not obvious, when written in this form, to see how to extract the physical contents, and how to generalize  $\mathcal{L}_{eff}^{\beta, \mu}$  to  $|\mu| \geq m$ , since then it appears to be divergent. In particular we notice that the high  $T$  behaviour given in [10] is not correct. After a Poisson resummation in  $l$ , rewriting the sum over  $l$  as a contour integral and carefully deforming the contours it is, however, possible to show that Eq.(2.14) is equal to Eq.(2.12) which, of course, is valid for all  $T$  and  $\mu$ .

### 3 The Physical Content of $\mathcal{L}_{eff}$

There are several dimensionful parameters related to  $\mathcal{L}_{eff}$ , i.e.  $T$ ,  $\mu$ ,  $m$ , and  $B$ , that can be large or small compared to each other. We shall only focus on a few of these limits which we think are particularly interesting.

The second term in Eq.(2.12) has an oscillatory behaviour that we can explore in the limit where  $\{T = 0, eB \ll \mu^2 - m^2 \ll m^2\}$ . This is a non-relativistic limit (in the sense that the kinetic energy is much smaller than  $m$ ) with a degenerate Fermi sea and a weak

external field. The oscillating part  $\mathcal{L}_{osc}$  of  $\mathcal{L}_1^{\beta,\mu}$  can be integrated in this approximation for which we obtain

$$\mathcal{L}_{osc} = -\frac{(eB)^{5/2}}{4\pi^4 m} \sum_{n=1}^{\infty} \frac{1}{n^{5/2}} \cos\left(\frac{\pi}{4} - n\pi \frac{\mu^2 - m^2}{eB}\right). \quad (3.1)$$

The oscillation frequency of this periodic function agrees with the one derived by Onsager [17] for the de Haas – van Alphen effect. Equation (2.12) describes the full relativistic generalization of this effect. The distance between the magnetic field of two adjacent minima of the magnetization is determined by

$$\left| \frac{1}{eB_i} - \frac{1}{eB_{i+1}} \right| = \frac{2\pi}{A}, \quad (3.2)$$

where  $A$  is the area of an extremal cross section of the Fermi sea.

In the limit of strong field,  $\{eB \gg T^2, m^2, \mu^2 - m^2\}$ , we can see from Eq.(2.13) that only the lowest Landau level contribute and  $\mathcal{L}_{eff}^{\beta,\mu}$  goes like a linear function of  $eB$ . We shall now reproduce this result from Eq.(2.12) and it turns out to be rather non-trivial. The leading  $B$  dependence in the first term in Eq.(2.12) is obtained by scaling out  $eB$  and taking  $eB \rightarrow \infty$  in the remainder. The total contribution is, apart from the thermal integration,

$$\frac{(eB)^{3/2}}{4\pi^{5/2}} \left[ \int_0^\infty \frac{dx}{x^{5/2}} (x \coth x - 1) - \sqrt{\frac{2}{\pi}} \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \right], \quad (3.3)$$

but this is actually identically zero. The next subleading term can be shown to be

$$\mathcal{L}_1^{\beta,\mu} = \frac{eB}{2\pi^2} \int_{-\infty}^{\infty} d\omega \theta(\omega^2 - m^2) f_F(\omega) \sqrt{\omega^2 - m^2}, \quad (3.4)$$

which is exactly the leading term from Eq.(2.13). This calculation shows that the oscillatory term in Eq.(2.12) is absolutely necessary to cancel the  $B^{3/2}$  term and to give the correct linear term.

Having shown that the thermal corrections in Eq.(2.12) are correct and comprehensible in physical terms, we now address the question of when they are important, i.e. when they dominate over the vacuum correction. For  $eB \gg m^2, T^2, \mu^2$ , the vacuum correction goes like

$$\mathcal{L}_1 \approx \frac{(eB)^2}{24\pi^2} \log\left(\frac{eB}{m^2}\right), \quad (3.5)$$

and it dominates over  $\mathcal{L}_1^{\beta,\mu}$ . However, when  $\{T = 0, eB \ll \mu^2 - m^2 \ll m^2\}$ , we have

$$\mathcal{L}_1 \approx \frac{(eB)^2}{360\pi^2} \left(\frac{eB}{m^2}\right)^2, \quad (3.6)$$

and

$$\mathcal{L}_1^{\beta,\mu} \approx \frac{(eB)^2}{12\pi^2} \log \left( \frac{|\mu|}{m} + \sqrt{\frac{\mu^2}{m^2} - 1} \right) \approx \frac{(eB)^2}{12\pi^2} \left( \frac{3\pi^2 n}{m^3} \right)^{1/3}, \quad (3.7)$$

where  $en$  is the charge density, and where we have neglected  $\mathcal{L}_{osc}$ . The density correction  $\mathcal{L}_1^{\beta,\mu}$  therefore dominates over  $\mathcal{L}_1$  when

$$\left( \frac{n}{m^3} \right)^{1/3} \gg \frac{1}{30(3\pi^2)^{1/3}} \left( \frac{eB}{m^2} \right)^2. \quad (3.8)$$

When  $T^2 \gg m^2 \gg eB$ , we have that

$$\mathcal{L}_1^{\beta,\mu} \approx \frac{(eB)^2}{24\pi^2} \log \left( \frac{T^2}{m^2} \right), \quad (3.9)$$

and we do not agree with the high temperature and weak field limit in [10]. (We notice the similarity of our result with  $\mathcal{L}_{eff}^0$  for  $eB \gg m^2$ .) In this case the thermal contribution  $\mathcal{L}_1^{\beta,\mu}$  dominates over  $\mathcal{L}_1$  as given by Eq.(3.6) when

$$\frac{T}{m} \gg \exp \left[ \frac{1}{30} \left( \frac{eB}{m^2} \right)^2 \right] \approx 1. \quad (3.10)$$

Another useful way of extracting the physical information from  $\mathcal{L}_{eff}$  is to define an effective coupling constant as [16, 18]

$$\frac{1}{\alpha(T, \mu, B)} = \frac{1}{\alpha} - \frac{1}{\alpha B} \frac{\partial \mathcal{L}_{eff}}{\partial B}, \quad (3.11)$$

in analogy with the definition of the renormalized coupling in the vacuum sector in connection with Eq.(2.9). Special care has to be taken when evaluating the derivative of the oscillating term in Eq.(2.12). In the limit when  $eB = 0$ , we obtain the effective coupling  $\alpha(T, \mu) = \alpha(T, \mu, B = 0)$  given by

$$\frac{1}{\alpha(T, \mu)} = \frac{1}{\alpha} - \frac{2}{3\pi} \int_{-\infty}^{\infty} d\omega \frac{\theta(\omega^2 - m^2)}{\sqrt{\omega^2 - m^2}} f_F(\omega). \quad (3.12)$$

When  $T = 0$ , we therefore get an effective coupling  $\alpha(\mu) = \alpha(T = 0, \mu)$  such that

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha} - \frac{2}{3\pi} \log \left( \frac{|\mu|}{m} + \sqrt{\frac{\mu^2}{m^2} - 1} \right). \quad (3.13)$$

In the limit  $\mu = 0$ , we find the following asymptotic behaviour of the corresponding effective coupling  $\alpha(T) = \alpha(T, \mu = 0)$ :

$$\frac{1}{\alpha(T)} = \frac{1}{\alpha} - \frac{4}{3\pi} \int_{\beta m}^{\infty} \frac{dx}{\sqrt{x^2 - (\beta m)^2}} \frac{1}{e^x + 1} \approx \frac{1}{\alpha} - \frac{2}{3\pi} \log \left( \frac{T}{m} \right), \quad (3.14)$$

for  $T \gg m$ . It is now clear that (only) for  $\mu \gg m$  and  $T \gg m$  the effective couplings  $\alpha(\mu)$  and  $\alpha(T)$  are solutions to the renormalization group equation (2.9) when  $\lambda$  is identified with  $\mu$  and  $T$  respectively (see in this context e.g. Refs.[19, 11]). We also note that Eq.(3.5) leads to an effective coupling  $\alpha(B) = \alpha(T = 0, \mu = 0, B)$  with an asymptotic behaviour

$$\frac{1}{\alpha(B)} \approx \frac{1}{\alpha} - \frac{1}{3\pi} \log\left(\frac{eB}{m^2}\right) \quad . \quad (3.15)$$

The effective coupling defined in Eq.(3.11) can also be extracted from the residue of the thermal Debye-screened photon propagator (see Ref.[19]).

We have only considered a few particular limits in this paper and there are many more to explore in different physical situations. All information needed to do that is contained in Eq.(2.12).

## 4 Conclusions

We have established the correct form of the one-loop QED effective action at finite temperature and density to all orders in a constant external magnetic field, and the result differs from earlier attempts. From the form of  $\mathcal{L}_{eff}^{\beta,\mu}$  presented in Eq.(2.12) we have checked several limits that can be understood from a physical point of view. A great advantage with our expression for  $\mathcal{L}_{eff}^{\beta,\mu}$  is that the thermal distribution function  $f_F(\omega)$  occurs explicitly. This means that it is easy to study other thermal situations by simply replacing  $f_F(\omega)$  with some other distribution.

The importance of the thermal correction depends on the value of  $B$ ,  $T$  and  $\mu$ . In many physically interesting cases they are all large compared to  $m$  and often of the same order of magnitude, which makes it difficult to obtain analytical approximations. It is, however, straightforward to use Eq.(2.12) for numerical calculations.

Even though the correction to the free energy is small compared to the value without the external field there are other quantities that are effected by the presence of the heat-bath. For instance, the magnetization of a degenerate Fermi sea as was briefly discussed in Section 3. One could also expect that QED radiative corrections at finite temperature and density and with the strong magnetic fields discussed in the Introduction could effect the electroweak transition rates, relevant for the Big-Bang primordial nucleosynthesis. We will return to this issue elsewhere.

We have, furthermore, calculated an effective coupling constant defined from the part of  $\mathcal{L}_{eff}^{\beta,\mu}$  which is quadratic in  $eB$ . It satisfies asymptotically a naive zero temperature



renormalization group equation where the renormalization scale is replaced by  $T$ ,  $\mu$  or  $\sqrt{eB}$ .

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